2F3-03

Description and Inference of Hybrid System on Situation Calculus

Katsunori NAKAMURA Akira FUSAOKA

Department of Computer Science Ritsumeikan University

A hybrid system is a dynamical system in which continuous and discrete dynamics are interacting each other. In this paper, we propose to use a nonstandard model of the situation calculus to deal with the hybrid system, in which the dynamics is described on the ontology of the hyperreals $^*\mathbf{R}$ rather than $\mathbf{R}$. On this framework, we disscuss about the inherent problems to the discrete dynamics such as Zeno problem.

1. INTRODUCTION

A hybrid system is a dynamical system in which continuous and discrete dynamics are interacting each other. A realtime system is usually hybrid because it often contains the digital controller for the continuous environments. In this paper, we propose to use a nonstandard model of the situation calculus to deal with the hybrid system, in which the dynamics is described on the ontology of the hyperreals $^*\mathbf{R}$ rather than $\mathbf{R}$. In comparison with other methods such as hybrid automata, this method has the following advantages:

(1) Since the continuous change is defined by the sequence of the actions with the infinitesimal effect in the very small duration, we can deal with both the continuous and discrete dynamics uniformly in the discrete but hyperfinite state transition paradigm.

(2) The completeness in the space of discrete and continuous dynamics is naturally introduced so that it allows to deal with the fixed point, for example, an asymptotic behavior toward limits. So-called Zeno is a typical problem about the fixed point that exhibits the infinite discrete value changes in the finite duration. Although it is often pointed out to be a significant problem in the modeling of hybrid system, any other method is weak at this point except [Zhang et al., 2000].

(3) Since the all theorems of the standard situation calculus hold even in its nonstandard model (the transfer principle [Robinson, 1974]), we can use the various properties of the discrete theory including the induction axiom even in its nonstandard extension [Reiter, 2001].

2. Nonstandard situation calculus NSC

We characterize a situation in the standard situation calculus **SSC** by a set of the independent fluent. Namely, a set of situation is :

$\mathbf{Sit} = \{< f_1, f_2, ..., f_m >| \ f_1 \in \mathbf{R}, f_2 \in \mathbf{R}, ..., f_m \in \mathbf{R}\}$.

We use the special fluent "time T". We denote the set of action by **Act**. Every action has a duration τ where $\tau(a, s) = T(do(a, s)) - T(s)$.

A situation in **NSC** is constructed from the those of **SSC** via the ultra product formation [Robinson, 1974].

Definition 1. Ultra filter: Let $\mathcal{F}$ be a family of the subsets of $\mathbf{N}$ which satisfy the following conditions, where $\mathbf{N}$ is the set of all natural numbers.

(1) $\mathbf{N} \in \mathcal{F}$, $\phi \notin \mathcal{F}$
(2) if $A \in \mathcal{F}$ and $A \subset B$ then $B \in \mathcal{F}$
(3) if $A \in \mathcal{F}$ and $B \in \mathcal{F}$ then $A \cap B \in \mathcal{F}$
(4) for any $A \subset \mathbf{N}$, $A \in \mathcal{F}$ or $\mathbf{N} - A \in \mathcal{F}$
(5) if $A \subset \mathbf{N}$ is finite then $\mathbf{N} - A \in \mathcal{F}$

$\mathcal{F}$ is called an ultra filter.

Definition 2. Hyperreal: We fix an ultra filter $\mathcal{F}$. Let W denote a set of sequences of real numbers $(a_1, a_2, \cdots)$. The hyperreal number $^*\mathbf{R}$ is defined by introducing the following equivalence relation into W

$$(a_1, a_2, \cdots) \sim (b_1, b_2, \cdots) \Leftrightarrow \{k \mid a_k = b_k\} \in \mathcal{F}$$

Namely, $^*\mathbf{R} = W/\sim$. We denote the equivalence class of $(a_1, a_2, \cdots)$ by $[(a_1, a_2, \cdots)]$. We define a relation $a \approx b$ if the distance from a to b is infinitesimal. We distinguish the nonstandard variables and function symbols from the standard one by attaching * to them, although it is omitted in the clear cases.

Definition 3. Nonstandard situation: We fix an ultra filter $\mathcal{F}$. The situation in **NSC** is defined by

$$^*\mathbf{Sit} = \{< [(f_1^1, f_1^2, \cdots)], [(f_2^1, f_2^2, \cdots)], \cdots, [(f_m^1, f_m^2, \cdots)] > \mid \{n \mid < f_1^n, f_2^n, \cdots, f_m^n > \in \mathbf{Sit}\} \in \mathcal{F}\}$$

Definition 4. Action and do function: The set of action **Act** is also enlarged to the set $^*\mathbf{Act}$ by

$$^*\mathbf{Act} = \{[(a_1, a_2, \cdots)] \mid a_i \in \mathbf{Act} \text{ for each } i\}$$

The *do* function is transferred to

$$^*do([(a_1, \cdots)], [(s_1, \cdots)]) = [(do(a_1, s_1), do(a_2, s_2), \cdots)]$$

3. The description of Hybrid system

A Hybrid system can be characterized generally by using a sequence $[a_1, a_2, \cdots, a_n]$ (n may be infinite). Many properties of dynamics can be represented in **NSC**. For example,

Continuity: A system $[a_1, a_2, \cdots, a_n]$ is continuous if and only if $\forall i, s[f(s) \approx f(do(a_i, s) \wedge \tau(a_i) \approx 0]$ for any fluent f.

Zeno: A system $[a_1, a_2, \cdots, a_n]$ is Zeno if and only if there exists an infinite subsequence $(i1, i2, \cdots, ik)$, $\forall i, s[f(s_{ij}) \not\approx f(do(a_i, s_{ij})) \wedge \Sigma_{ik}\tau(a_{ik})$ is finite $]$ for some fluent f.

Repelling, Attractive: The situation s is repelling related to a, f if $\forall s'[s' \approx s \supset f(s') < f(do(a, s'))]$.

A: fusaoka@cs.ritsumei.ac.jp, katsu@muse.cs.ritsumei.ac.jp

The situation s is attractive related to a, f if
$\forall s'[s' \approx s \supset f(s') \geq f(do(a, s'))]$.
In order to deal with the hybrid system actually, we need the following inferential device :
(1) a reasoning system for the situation calculus
(2) an extended arithmetic for $^*\mathbf{R}$
(3) a set of transfer rules between $^*\mathbf{R}$ and $\mathbf{R}$ such as $(1 - \frac{1}{n})^n \approx e^{-1}$ if n is infinite.

4. Examples

Water tankI: Consider a water tank with a tap in the bottom of the tank. Assume that the tap discharges the water at a rate proportional to the level of water x in the tank, the physical scenario is given by the differential equation $\frac{dx}{dt} + kx = 0$ if k is constant.
This dynamics is described by the sequence of situation $(s_0, s_1, \cdots,)$ generated by iteration of the infinitesimal action ∂a with the duration τ where for each s_n satisfies
$x(^*do(\partial a, ^*s_n)) = {}^*x(^*s_n) - k\frac{t}{n}{}^*x(^*s_n)$
This difference equation is solved by simple calculation. As a result, we can obtain the accurate situation of the water tank after t seconds. Namely, for the very large n such that $t = n\tau$
${}^*x(^*s_n) = x(s_0)(1 - k\frac{t}{n})^n$

We can find out the standard value near the above equation by the transfer rule. Namely, $x(t) = x(s_0)e^{-kt}$.

Water tankII: Consider a coupling of two water tanks. Let x, y denote the level of water in Tank A and Tank B. This time, we assume that the tap in the bottom of each tank discharges the constant flow q of the water. Also the constant flow denoted by p of the water is poured exclusively to either Tank A (we call the state A) or Tank B (state B) at each time. We use the control strategy

if $state = A \wedge x \geq h \wedge y < h$ **then** *switchto B*
if $state = B \wedge x < h \wedge y \geq h$ **then** *switchto A*

The physical scenario is given by a alternating sequence of two actions a : pouring water in Tank A and b : pouring water in Tank B. This model contains two behaviors peculiar to hybrid dynamics. After the infinite iteration of a, b, both tanks approach to the level h simultaneously (Zeno point), and then it repeats the actions a, b forever but we need the infinitesimal analysis to predict the situation at each time because it depends on the very small fluctuation at the fixed point (Repelling point).

[Zeno point] The action a, b are defined by:
$x(do(a, s)) = x(s) + (p - q)\frac{y(s)-h}{q}$, $y(do(a, s)) = h$
$y(do(b, s)) = h$, $x(do(b, s)) = y(s) + (p - q)\frac{x(s)-h}{q}$
Let $s_{2n+1} = do(a, s_{2n})$, $s_{2n} = do(b, s_{2n-1})$ and $x(s_0) = y(s_0) = c$. By the simple calculation, the duration of the actions are given by,
$\tau(a, ^*s_n) = \tau(b, ^*s_n) = (\frac{p-q}{q})^{n-1}\frac{p(c-h)}{q^2}$
The zeno point $^*s_n$ is defined by
$x(^*s_n) \approx y(^*s_n) \approx h$
$T(^*s_n) = \frac{c-h}{q(2q-p)}\{2q - p(\frac{p-q}{q})^n\}$
Because n is infinite, we have the standard Zeno point at the time $T(s) = \frac{2(c-h)}{2q-p}$

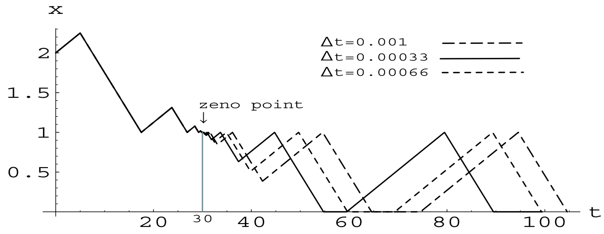

Figure 1: The orbit of the level of Tank A

[Repelling point] This Zeno point is repelling for $t > T(s)$ so that the behavior of the system depends on the situation immediately after the Zeno point. We introduce the quantum duration Δt in **NSC** which is informally corresponding to the sampling rate of the hybrid system. Δt is infinitesimal and the duration of any action should not be smaller than Δt. Namely, we consider a time that is complete in the standard sense but is discrete and even not dense in the nonstandard sense. The Figure 1 shows the influence of Δt for the orbit of the tanks after the Zeno point where we use a small number as Δ. The reasoning method to jump out from the fixed point is simple due to this quantum time. We just use Δt instead of $\tau(a, s)$ whenever $\tau(a, s) < \Delta t$ and determine the situation immediately after the fixed point. Namely, we can select the state A or B for the next situation to the Zeno point by comparing $\tau(a, ^*s_{2n})$ and $\tau(b, ^*s_{2n-1})$ to the given Δt, and evaluate the level of each tank. Assume that the levels of tank A,B are $h - \delta$,h respectively and we are in the state A immediately after the Zeno point. Then the behavior of each tank is given by
$x(s_{2n}) = h - \delta(\frac{q}{p-q})^{2n}$, $y(s_{2n-1}) = h - \delta(\frac{q}{p-q})^{2n-1}$

5. Concluding Remarks

One direction for future research will be a nonstandard treatment of concurrent and continuous actions of multiagent such as the differential game.

References

[Robinson, 1974] Robinson, A. 1974. *Non-standard analysis,* Amsterdam: North Holland.

[Reiter, 2001] Reiter, R. 2001. *Knowledge in Action,* Cambridge: The MIT press.

[Zhang et al., 2000] Zhang, J., et al. 2000. Dynamical System Revisited: Hybrid Systems with Zeno Execution, *Proc.HSCC2000. LNCS 1790*,451-464.

[Iwasaki et al., 1975] Iwasaki, Y.et al. 1995. Modeling Time in Hybrid System: How Fast Is "Instantaneous"?, *Proc.IJCAI-95.*

[J.M.Davoren et al., 2000] J.M.Davoren et at. 2000, Logics for Hybrid Systems, *PROCEEDINGS OF THE IEEE*,Vol.88 No.7,July 2000,985-1010

[A.Fusaoka et al., 2002] A.Fusaoka et al.,2002 On A Nonstandard Model of Situation Calculus, *Journal of Japanese Society for Artificial Intelligence* Vol.17 No.5, J-STAGE 557-564 (in Japanese)